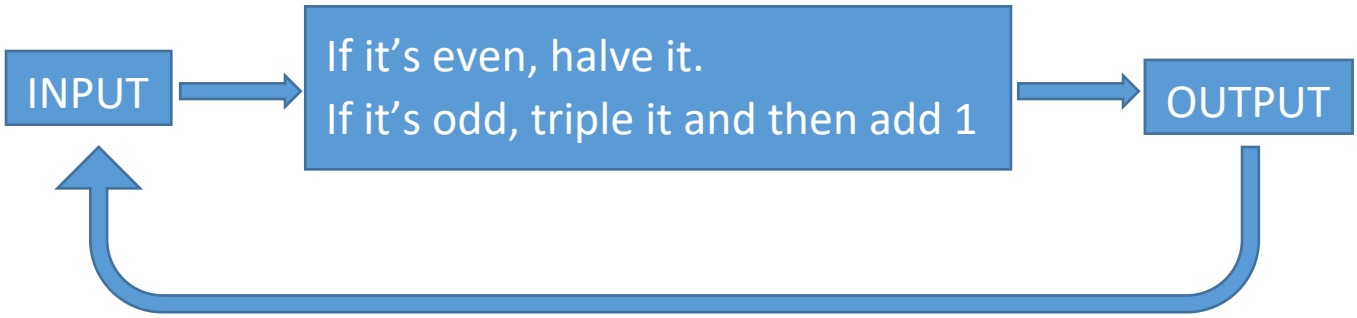
Collatz Conjecture

1. Choose a starting number (It must be a positive whole number)

   Follow this process:



   As an example, if we started with 5, the sequence we get is:

$$5 \to 16 \to 8 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1 \to 4 \to 2 \to 1 \; etc.$$

   Work out the sequence formed from the following starting numbers:

   a) 3
   b) 10
   c) 7
   d) A positive whole number of your choice
   e) A different number of your choice: as large as you dare!

2. Further questions to investigate:

   a) Which single digit positive number takes the longest to get to 1?
   b) Which type of numbers tend to get to 1 quite quickly?
   c) Will all starting positive whole numbers get to 1?
   d) Can you find the 28 positive whole numbers that take 10 steps or fewer to get to 1?

Collatz Conjecture - Answers and Further Thoughts

1.

a) $3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$

b) Start at 10 in the picture above.

c) $7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26 \to 13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$

2. Further questions to investigate:

a) 9 takes the longest. Starting at 9, we reach 1 in 19 steps. (The first 1 is the 20th number in the sequence.)

b) Powers of 2 get to 1 quickly, because no *tripling and adding 1* takes place.

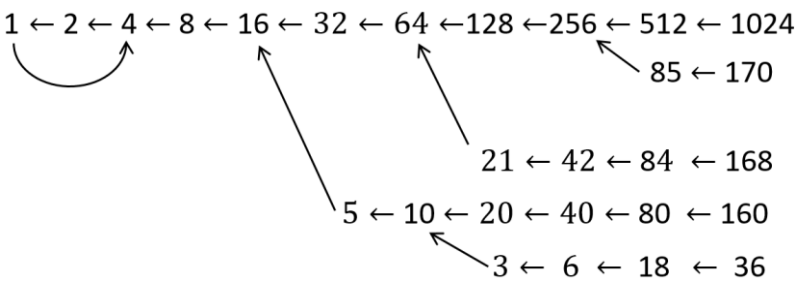
c) Will all starting positive whole numbers get to 1?

Amazingly, nobody knows the answer to this question. No one has yet found a number that doesn't eventually get to 1, but no one has been able to prove that it's impossible.

Perhaps you will be the first person to find such a number, or *prove* that all starting numbers will lead to 1.

If you do, you will have solved the *Collatz Conjecture* (A *conjecture* is a statement that is thought to be true, but that no one has been able to prove yet. Once it's been proved it becomes a *Theorem.*)

d) Building a *tree* from 1 is a nice way to see the numbers that get to 1 quickly:



To find out more, look up
Collatz Conjecture
Collatz Conjecture in Color