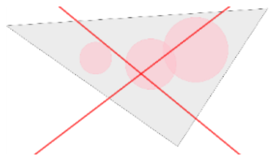
Malfatti's Problem

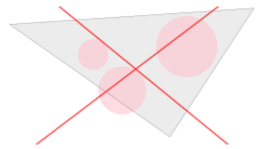
Becoming intelligent is partly about becoming good at telling the difference between the *true* and the *seemingly true.*

A nice example from Maths of an widely accepted truth being shown to be wrong years later is *Malfatti's Problem.*

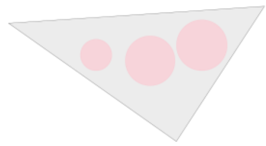
The puzzle is this: Given a triangle, what is the best way to insert 3 non-overlapping circles into the triangle in order to make the combined areas of the circles as large as possible?



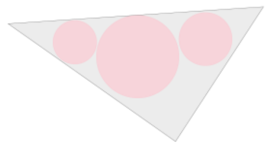
Invalid attempt (Circles overlap)



Invalid attempt (Circles not all inside triangle)



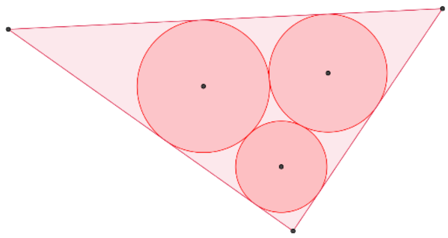
Valid attempt



Better valid attempt (Larger circle area)

As a motivation for the puzzle, you could imagine the triangle to be a cross-section (slice) of a slab of marble from which we want to cut 3 cylindrical columns for a grand building. We want to waste as little marble as possible.

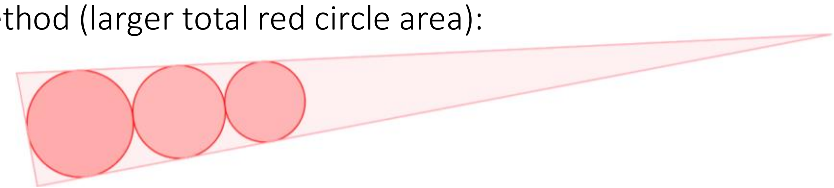
Malfatti became associated with this problem in the early 1800s, after deciding (wrongly as it turns out) that the way to get the largest circle area was by making sure that each circle touched two of the triangle's side, as in the picture below:



But, even though this seemed to be sensible, a long thin triangle shape reveals a way to get more circle area than using Malfatti's method:

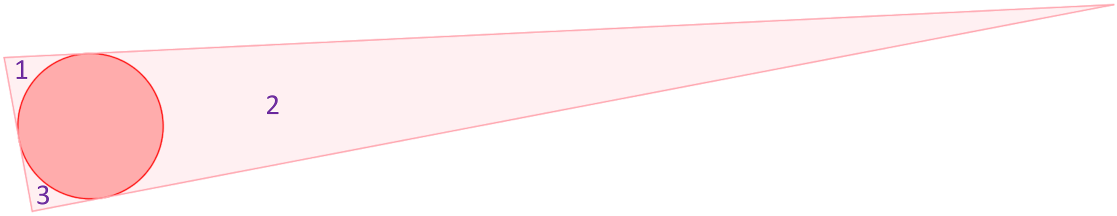
Malfatti's Method:

A better method (larger total red circle area):

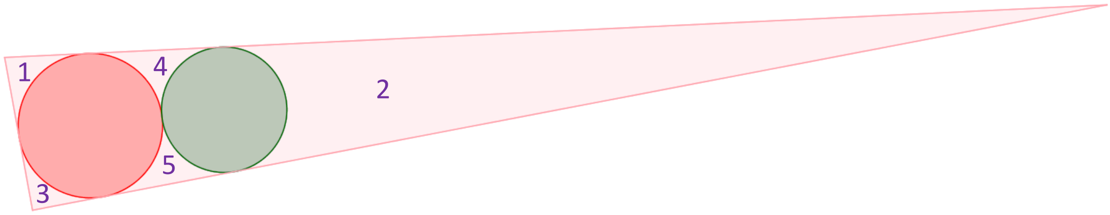


You can get to these three circles in the better method by following the following *algorithm* (set of instructions):

1. Insert into the triangle the circle that fills as much area as possible:



2. Inscribe a circle as big as possible into one of the three remaining gaps (numbered in the previous picture).



3. Inscribe a circle as big as possible into one of the five remaining gaps (numbered in the previous picture).

So for this long thin triangle at least, Malfatti's method isn't the best; this new algorithm (which is called a *Greedy Algorithm*, because each new circle greedily *grabs* as much space as it can) is better.

But it gets even worse for Malfatti! In the 1960s, it was proved that the *Greedy Algorithm* is better than Malfatti's method not just for one triangle, or some triangles, but for *every possible triangle.* There is no possible triangle for which Malfatti's idea is better than the *Greedy Algorithm*

So, for Malfatti at least, there was a huge difference between *true* and *seemingly true*. But will lovers of the *Greedy Algorithm* fall into the same trap? Or is this definitely the most efficient method?

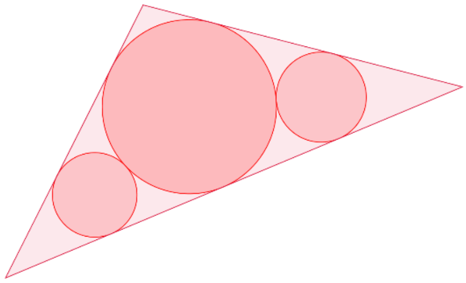
Well, It has now been proved that indeed the *Greedy Algorithm* **will** always maximise the total area of the 3 circles. However, whether this extends to trying to get 4 or more circles out of a triangle is (at time of writing) an unsolved problem. Maybe *you* might solve it one day.

Malfatti's Problem: Comprehension Questions:

1. Here are 3 circles inside a triangle.

   Which method has been used to create them?

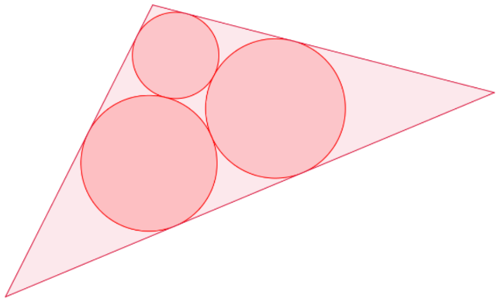
   A. Malfatti's method
   B. The Greedy Algorithm
   C. Neither of the above.



2. Here are 3 circles inside a triangle.

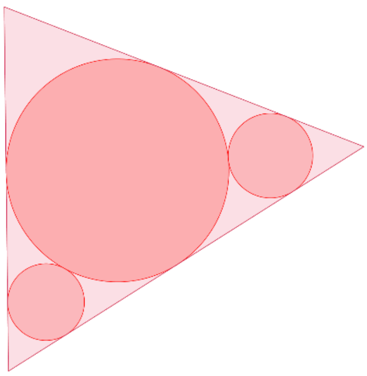
   Which method has been used to create them?

   A. Malfatti's method
   B. The Greedy Algorithm
   C. Neither of the above.

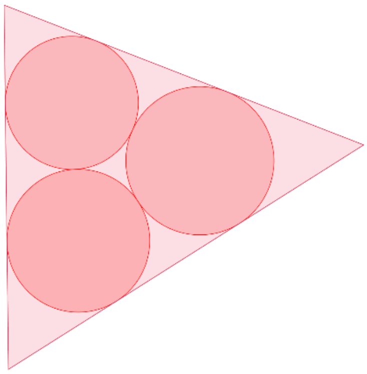


3. Here are two pictures of circles inside two copies of the same triangle.

   <u>Picture 1</u>



   <u>Picture 2</u>



   For which picture is the area of the circles larger?

   (You don't have to do any measurement; the answer to this question can be found in the article)

Malfatti's Problem: Comprehension Answers:

1. B) The Greedy Algorithm

2. A) Malfatti's Method

3. The Greedy Algorithm always produces a greater total circle area than Malfatti's, so the total circle area is greater in Picture 1.