Fermat's Last Theorem

> *"I have discovered a truly remarkable proof of this theorem which this margin is too small to contain."*
> [Pierre de Fermat, 1637, in the margin of a Maths Textbook about Number Theory]

French mathematician (well, actually, he was a lawyer for his day job), Pierre de Fermat, was claiming that he had discovered (but didn't publish) a proof of the result that has come to be known as *Fermat's Last Theorem,* one of the most famous bits of Maths in modern History.

To help to understand what it is, we'll introduce power notation:

Squares, Cubes, 4th Powers, etc.

For example:

$5\ squared = 5^2 = 5 \times 5 = 25$

$5\ cubed = 5^3 = 5 \times 5 \times 5 = 125$

$5\ to\ the\ power\ of\ 4 = 5^4 = 5 \times 5 \times 5 \times 5 = 625$

and so on.

Fermat's Last Theorem is all about trying to find positive whole numbers that can fill in the boxes in equations such as these:

$\square^2+\square^2=\square^2$ or $\square^3+\square^3=\square^3$ or $\square^4+\square^4=\square^4$ etc.

For the first of these, we can find numbers to go in the boxes, for example:

$$3^2 + 4^2 = 5^2$$

But it seems more difficult to find numbers to fill the boxes in the '3-version' or the '4-version', or any others, except the '2-version'.

In fact, *Fermat's Last Theorem* is the statement that it is *only possible* to fill in the gaps in the '2-version'. That is to say that, no matter how hard we look, we will never find positive whole numbers that will satisfy $\square^3+\square^3=\square^3$ or $\square^4+\square^4=\square^4$ or any such equation (replacing the 4 in the power by any whole number larger than 2).

Since Fermat proposed this idea, and stated that he had worked out a proof of it, many mathematicians have tried, and failed, to work out a proof.

Fermat himself did at least write down a correct proof that for the '4-version' ($\square^4 + \square^4 = \square^4$; you cannot find positive whole numbers to go in the boxes). But, as was typical of Fermat, he didn't write down the proof that he claimed he had for his complete theorem.[1]

For many years, mathematicians chipped away at this theorem. To cut a long story short, by 1980, mathematicians knew that the following equations could not be filled in with positive whole numbers:

$$\square^3 + \square^3 = \square^3$$

$$\square^4 + \square^4 = \square^4$$

$$\vdots$$

$$\square^{125000} + \square^{125000} = \square^{125000}$$

Impressive as this is, if we regard this as 124998 equations proved to be impossible, it's 124998 of *infinitely many* that need to be proved to be impossible before Fermat's Last Theorem is completely proved. So, in a way, these 124998 separate results aren't even close to the desired result.

A mathematician, Andrew Wiles, first heard about Fermat's Last Theorem when he was a child (perhaps like you, reading this!). In 1994, when he had learnt much more Maths, and then worked on something called the *Taniyama-Shimura Conjecture*, in secret, for over 7 years, he finally solved Fermat's Last Theorem.

The *Taniyama-Shimura Conjecture* was a powerful and complicated mathematical result, which has many consequences. Fermat's Last Theorem is a relatively small consequence of *Taniyama-Shimura.* We could think of this as the mathematical equivalent of using a sledgehammer (Taniyama-Shimura) to crack a nut (Fermat's Last Theorem).

Back in the 1600s, Fermat did not have access to this sledgehammer, so the mystery remains over whether Fermat did, in fact, have a 'truly remarkable proof' of this theorem. We'll probably never know.

[1] Fermat wrote down many unproved 'theorems' about numbers that, since his death, other mathematicians have worked on. All but two of them were proved to be correct (there are known proofs). On one result, it turned out that Fermat was mistaken. The only result that eluded mathematicians for centuries was the one this article is about. This is why it became known as Fermat's *Last* Theorem.

Fermat's Last Theorem: Comprehension Questions

1. Find the missing numbers in these equations:

   a) $5^2 + 12^2 = \square^2$

   b) $6^2 + \square^2 = 10^2$

   c) $\square^2 + 24^2 = 25^2$

2. On a rainy day, Peter is entertaining himself by trying to find 3 positive whole numbers that can fit in the boxes in this equation:

$$\square^4 + \square^4 = \square^4$$

   Having read this article, you should now know that Peter's task is impossible.

   Which mathematician is credited with having first proved that Peter's task is impossible?

3. On an episode of *The Simpsons*, Homer Simpson has written the following on a blackboard:

$$3987^{12} + 4365^{12} = 4472^{12}$$

   How do we know that this equation must be wrong?

Fermat's Last Theorem: Comprehension Questions

4. Find the missing numbers in these equations:

   a) $5^2 + 12^2 = \square^2$

   b) $6^2 + \square^2 = 10^2$

   c) $\square^2 + 24^2 = 25^2$

5. On a rainy day, Peter is entertaining himself by trying to find 3 positive whole numbers that can fit in the boxes in this equation:

$$\square^4 + \square^4 = \square^4$$

   Having read this article, you should now know that Peter's task is impossible.

   Which mathematician is credited with having first proved that Peter's task is impossible?

6. On an episode of *The Simpsons*, Homer Simpson has written the following on a blackboard:

$$3987^{12} + 4365^{12} = 4472^{12}$$

   How do we know that this equation must be wrong?

Fermat's Last Theorem: Comprehension Answers

1. Find the missing numbers in these equations:

    a) $5^2 + 12^2 = \mathbf{13}^2$

    b) $6^2 + \mathbf{8}^2 = 10^2$

    c) $\mathbf{7^2} + 24^2 = 25^2$

2. On a rainy day, Peter is entertaining himself by trying to find 3 positive whole numbers that can fit in the boxes in this equation:

$$\square^4 + \square^4 = \square^4$$

Having read this article, you should now know that Peter's task is impossible.

Which mathematician is credited with having first proved that Peter's task is impossible?

Pierre de Fermat

3. On an episode of *The Simpsons*, Homer Simpson has written the following on a blackboard:

$$3987^{12} + 4365^{12} = 4472^{12}$$

How do we know that this equation must be wrong?

If this equation is wrong, then it would go against Fermat's Last Theorem, which (since 1984) has been known to be correct.

[You might like to type into your calculator $3987^{12} + 4365^{12}$ and then 4472^{12} and see what happens. If your calculator is like mine, it seems that they are equal. Fear not. This article has not lied to you. They only *appear* to be equal because the numbers are so big. If you know a trick to work out whether a number is divisible by 3 or not, you might be able show that $3987^{12} + 4365^{12}$ is a multiple of 3, but 4472^{12} is not. So they can't be equal.]

To find out more, look up *Fermat's Last Theorem*, or read:
pages 50 to 58 of *Professor Stewart's Cabinet of Mathematical Curiosities* by Ian Stewart
or, for a more lengthy read, *Fermat's Last Theorem* by Simon Singh