

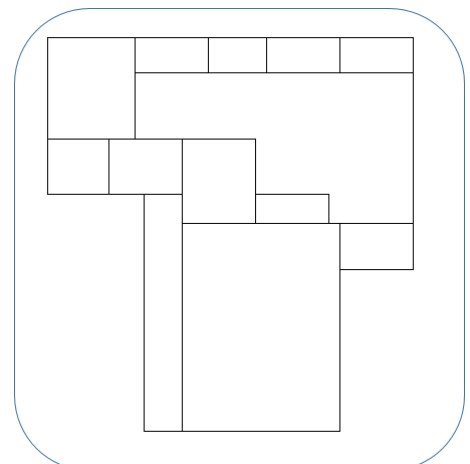
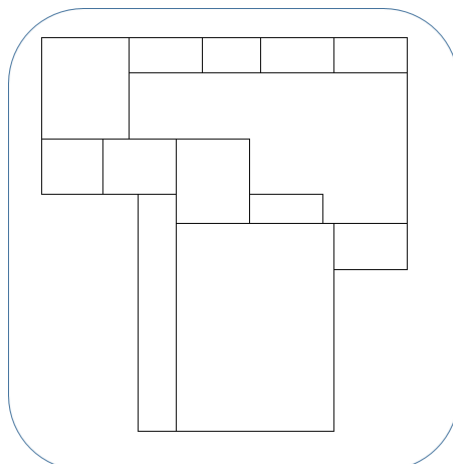
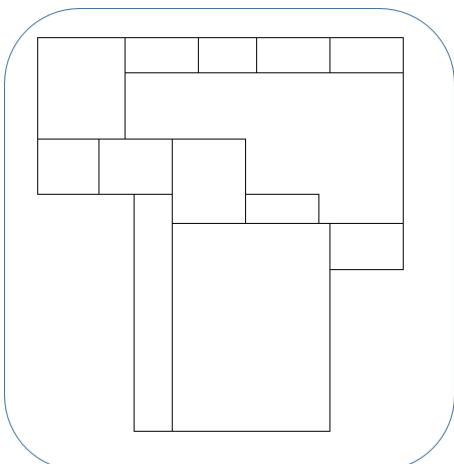
Colouring Maps

Below (on the left) is a political map of South America (correct at time of writing), with each country in a different colour. No two countries that share a border are the same colour. For this task, we're interested in whether we can still achieve this with fewer colours. On the right is a simplified diagram. Information about which countries have borders with which others has been preserved, and is easier to see. Other information has been lost, which is OK for our purpose.



The map on the right has been coloured with just 6 different colours. (I'm including *the sea* as a region in my map. Note that I'm allowing a country to have the same colour as the sea, as long as it doesn't touch it.) I have achieved my objective that no two regions that share a border are the same colour. Your challenge is to achieve this objective using fewer colours.

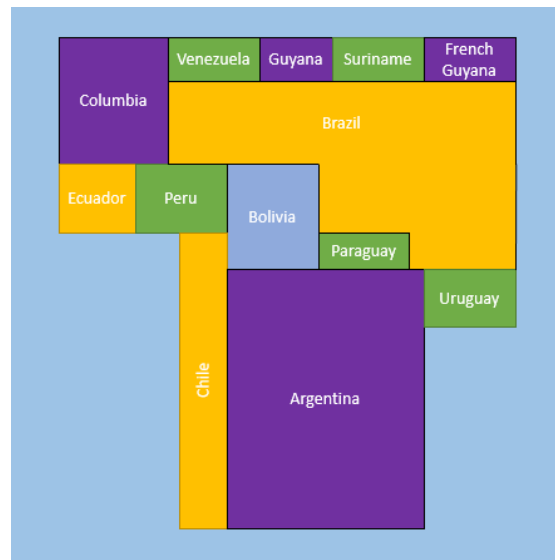
1. Try just using 5 colours (including the sea). Can you do it? If you think it's impossible, can you *prove* it?
2. Try just using 4 colours (including the sea). Can you do it? If you think it's impossible, can you *prove* it?
3. Try just using 3 colours (including the sea). Can you do it? If you think it's impossible, can you *prove* it?



Colouring Maps: Answers and Further Thoughts

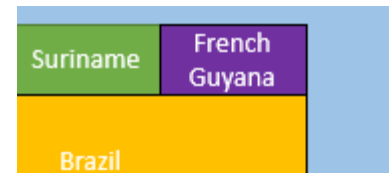
It is possible to do it using 4 colours: (See below)

It's therefore also possible to do it using 5 (just pick a country and colour it, for example, red).



It is **impossible** to manage it using just 3 colours.

We can see that it's impossible just by looking at this top right section, containing 1) Suriname, 2) French Guyana, 3) Brazil, and 4) The sea.



Each of these four regions touches each of the other 3, which means that they have to be all different colours.

So we can colour South America and its adjacent sea using only 4 different colours (but we can't with just 3 different colours).

The question remains: Can we colour *every possible map* (Europe? Africa? A made up one?) in this way using only 4 different colours? There are infinitely many possible maps, so this question seems very difficult to answer.

Amazingly, mathematicians have worked out that we **can**. This result is called the ***Four Colour Theorem***.

It was proved in 1976 Kenneth Appel and Wolfgang Haken (developing the work of others)

Their proof used complex mathematical ideas and logic to, somehow, reduce the number of possible maps that needed to be tested from *infinitely many* to under 2000. (It's pretty amazing that they were able to do this, I think). The rest of the proof involved finding a way to use 4 colours for each of these 2000ish possibilities. In a novel way at the time, they programmed a computer to help them with this second part of the task. Due to the limited computing power at the time, it took the machine one thousand hours to find and check them.

To find out more, look up: *The Four Colour Theorem*