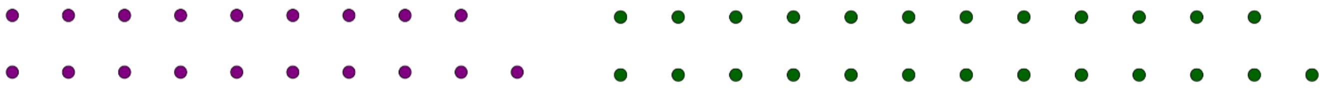
Introduction to mod 3

Paul says that the sum of the green dots and the purple dots in the picture below is 43.



1. Without counting the dots, how can you tell that Paul is wrong?

2. Complete the following, by writing either *even* or *odd* in the gaps

   a. $even\ number\ +\ even\ number\ =\ \_\_\_\_\_number$

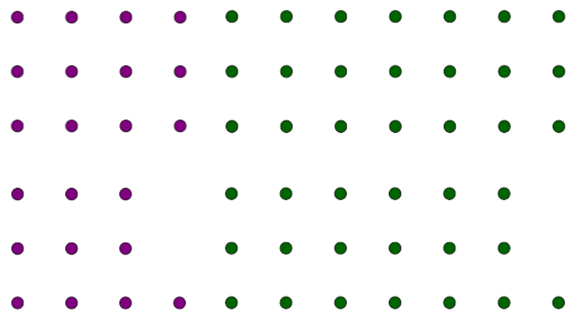
   b. $odd\ number\ +odd\ number\ =\_\_\_\_\_number$

   c. $even\ number\ +\ odd\ number\ =\ \_\_\_\_\_number$

These rules are surprisingly useful in Maths. We can make some rules like this about divisibility by 3 rather than divisibility by two, by first realising that every whole number falls into exactly one of three columns in the following table:

1 more than a multiple of three	*2 more than a multiple of three*	*multiple of three*
1 4 7 10 ⋮ (and so on)	2 5 8 11 ⋮ (and so on)	3 6 9 12 ⋮ (and so on)

3. If you add together two numbers in the *multiple of three* column, which column will the answer be in? (The picture might help). Will this always happen?



4. If you add together two numbers in the *1 more than a multiple of three* column, which column will the answer be in? (you can think about rearranging the dots in the picture) Will this always happen?

5. Figure out what happens for the other possibilities (Fill the gaps with <u>*1 more than a multiple of three*</u>, or <u>*2 more than a multiple of three*</u>, or <u>*multiple of three.*</u>)

   a. *2 more than a multiple of three* + *2 more than a multiple of three* =_______________

   b. *multiple of three* + *1 more than a multiple of three* =_______________

   c. *multiple of three* + *2 more than a multiple of three* =_______________

   d. *1 more than a multiple of three* + *2 more than a multiple of three* =_______________

6. Investigate what happens when you *multiply* a number in the *2 more than a multiple of three* column by a number in the *1 more than a multiple of three* column. Will your answer always be in the same column whenever you do this?
7. Investigate the other possibilities for multiplying numbers in different columns together.

Worksheet created by Tim Elton, 2020

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Introduction to mod 3: Answers and Further Thoughts

1. We can see that we'll get an even number of dots in total. 43 is not even, so Paul must be wrong.
2. Complete the following, by writing either *even* or *odd* in the gaps

   a. $even\ number\ +\ even\ number\ =\ even\ number$

   b. $odd\ number\ +odd\ number\ =\ even\ number$

   c. $even\ number\ +\ odd\ number\ =\ odd\ number$

3. *A multiple of three* + a *multiple of three* will always be a *multiple of three*.
4. *1 more than a multiple of three* + *1 more than a multiple of three* will always be *2 more than a multiple of three.*
5.

a) *2 more than a multiple of three* + *2 more than a multiple of three* = *1 more than a multiple of three*
b) *multiple of three* + *1 more than a multiple of three* = *1 more than a multiple of three*
c) *multiple of three* + *2 more than a multiple of three* = *2 more than a multiple of three*
d) *1 more than a multiple of three* + *2 more than a multiple of three* = *multiple of three*

Now that we know that it doesn't matter *which* number we choose from each column; we always get the same column for the answer, we can summarise the results we've seen in a neat operation table. (The numbers in the column and row headers refer to how many more than a multiple of three our number is.

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

This operation we've been doing is called *addition modulo 3.* So we can call the table of results above the *modulo 3 addition table.*

6. *2 more than a multiple of three* × *1 more than a multiple of three* will always be *2 more than a multiple of three.*

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

To find out more:
Investigate *modulo 4* and *modulo 5* addition and multiplication tables. There's something noticeable about how the *modulo 4* multiplication table behaves, compared to the others.
Or/and look up: *modular arithmetic*