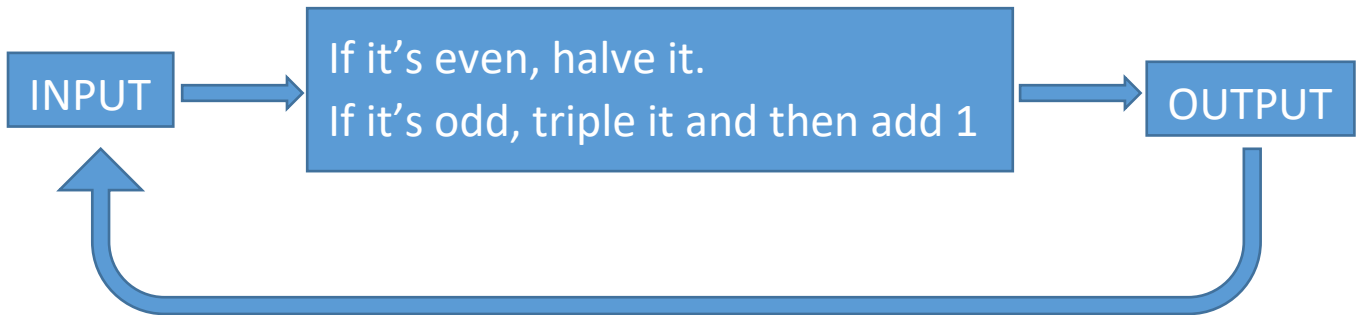


## Collatz Conjecture

1. Choose a starting number (It must be a positive whole number)

Follow this process:



As an example, if we started with 5, the sequence we get is:

$$5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1 \text{ etc.}$$

Work out the sequence formed from the following starting numbers:

- 3
  - 10
  - 7
  - A positive whole number of your choice
  - A different number of your choice: as large as you dare!
2. Further questions to investigate:
    - Which single digit positive number takes the longest to get to 1?
    - Which type of numbers tend to get to 1 quite quickly?
    - Will all starting positive whole numbers get to 1?
    - Can you find the 28 positive whole numbers that take 10 steps or fewer to get to 1?

## Collatz Conjecture - Answers and Further Thoughts

1.

a)  $3 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4$   
 $1 \leftarrow 2$

b) Start at **10** in the picture above.

c)  $7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4$   
 $1 \leftarrow 2$

2. Further questions to investigate:

a) 9 takes the longest. Starting at 9, we reach 1 in 19 steps. (The first 1 is the 20th number in the sequence.)

b) Powers of 2 get to 1 quickly, because no *tripling and adding 1* takes place.

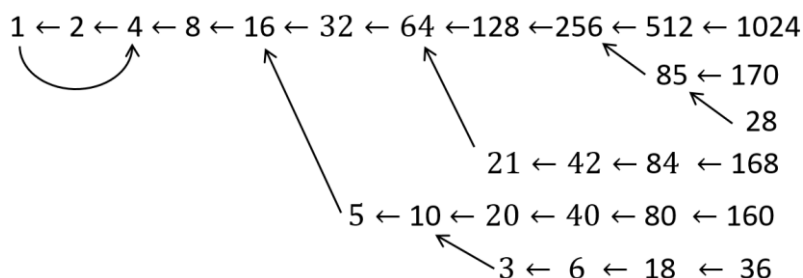
c) *Will all starting positive whole numbers get to 1?*

**Amazingly, nobody knows** the answer to this question. No one has yet found a number that doesn't eventually get to 1, but no one has been able to prove that it's impossible.

Perhaps **you** will be the first person to find such a number, or *prove* that all starting numbers will lead to 1.

If you do, you will have solved the *Collatz Conjecture* (A *conjecture* is a statement that is thought to be true, but that no one has been able to prove yet. Once it's been proved it becomes a *Theorem*.)

d) Building a *tree* from 1 is a nice way to see the numbers that get to 1 quickly:



To find out more, look up  
*Collatz Conjecture*  
*Collatz Conjecture in Color*