Unlimited 3s and 7s

Suppose you have an unlimited supply of 3s and an unlimited supply of 7s. We are interested in which whole number totals are possible (just by adding threes and sevens).

For example, 13 is possible, because $13 = 3 + 3 + 7$.

1. Find out which of the following totals are possible:

$1 =$	
$2 =$	
$3 =$	
$4 =$	
$5 =$	
$6 =$	
$7 =$	
$8 =$	
$9 =$	
$10 =$	
$11 =$	
$12 =$	
$13 =$	$3 + 3 + 7$
$14 =$	

2. You should have found that $12, 13$ *and* 14 are all possible totals. This means that we can get totals of $15, 16$ *and* 17 without much more work. How?

3. What is the largest whole number total that is impossible to get?

4. Now suppose that instead of unlimited 3s and 7s, we have unlimited 4s and 6s. Which totals are impossible to get?

5. Now suppose that it's unlimited 10s and 9s that we have. Can you work out which is the largest total that is impossible to get? (The number square picture may help.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Unlimited 3s and 7s - Answers and Further Thoughts

1.

$1 =$	(impossible)	$8 =$	(impossible)
$2 =$	(impossible)	$9 =$	$3 + 3 + 3$
$3 =$	3	$10 =$	$3 + 7$
$4 =$	(impossible)	$11 =$	(impossible)
$5 =$	(impossible)	$12 =$	$3 + 3 + 3 + 3$
$6 =$	$3 + 3$	$13 =$	$3 + 3 + 7$
$7 =$	7	$14 =$	$7 + 7$

If you know the scoring of the game of Rugby Union, you may have noticed that this is all about the possible scores achievable with converted tries (7 points) and penalties or drop-kicks (3 points).

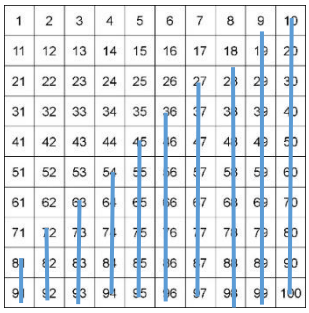
2. For 15, do whatever you did for 12 , then add a 3.
   For 16, do whatever you did for 13 , then add a 3.
   For 17, do whatever you did for 14 , then add a 3.

3. Extending the argument in 2, we can see that any total above 11 is possible, by adding as many 3s as you need.

   11 is not possible. It's not a multiple of 3, so you can't get there with just $3s$. If you use one 7 then this won't work, because all of 7, $7 + 3$, $7 + 3 + 3$, $7 + 3 + 3 + 3$ $etc.$ are 1 more than a multiple of 3. 11 is not. If we use 2 or more 7s then the total is already too large.

   So 11 is the largest number that is not possible.

4. 4 and 6 have common factor of 2, so only even numbers are possible. So all odd numbers are impossible, and so is 2.71 is the largest total impossible to get with 9s and 10s. The number grid below crosses out all possible totals up to 100



To find out more, investigate more pairs of numbers (that don't share a common factor). The ultimate challenge is to work out a formula that works out the largest total that is impossible to get, given a pair of numbers.

If you think you've found a formula, test it on the information you have already (11 for 3s and 7s. 71 for 9s and 10s.) Feel proud of yourself if it works for these!

If it does, the next step would be to *prove* that your formula will always work! That's not easy. Simply *testing* your formula on some pairs of numbers doesn't prove that it *always* works.

To find out more, look up: *Frobenius Numbers* and possibly *McNugget Numbers*.