Digit Multiplication Sequences

1 Follow this process:

> Write down a large whole number.
> *(to make it interesting, avoid 0 and 5 in your number)*
>
> Multiply the digits of your number together (you may use a calculator).
>
> Write down this new number.
>
> Multiply the digits of this together (you may use a calculator).
>
> Write down this new number.
>
> Keep on repeating this until you can go no further.

2 With your example, you should have found that you eventually (or quite quickly) got to a single digit. How many times did you multiply digits before you got there?

(In other words, How many *iterations* (*steps*) before you get to a single digit?)

3 What will happen if you follow this process with a starting number that has a 0 in it?
investigate!

4 What will happen if you follow this process with a starting number that has a 5 in it?
investigate!

5 By choosing a suitable starting number, how large can you make the *number of iterations (steps)?* Keep aiming to find a starting number that *beats your personal best.*

Digit Multiplication Sequences - Answers and Further Thoughts

1 The sequence would be different for your choice of starting number. Here's an example sequence, with the starting number as 2311983:

$$2311983 \rightarrow 1296 \rightarrow 108 \rightarrow 0$$

2 For my example, there were 3 iterations (arrows) before reaching a single-digit number, in my case, 0.

3 Any number with a 0 in it will immediately be followed in the sequence by 0.

4 If there is an even number anywhere in the rest of the number, (e.g. 3578) , then the next number will be a multiple of 10, (e.g. $3578 \rightarrow 840$). This new number has a zero in it, so we know what will happen next $(3578 \rightarrow 840 \rightarrow 0)$.

If there are only odd numbers along with the 5, (e.g. 35779) then the next number will be a multiple of 5 that is not a multiple of 10, so will end in a 5 (e.g. $35779 \rightarrow 6615$). Then we're at the same situation of our new number definitely having a 5 in it. Probably this new number has an even number in it (and you can look at the previous paragraph for what happens next).

5 Well done if you found a sequence with any more than the 3 iterations of my example.

You'll probably have found that most numbers go to a single digit (typically 0) extremely quickly.

8867 is quite interesting, needing 6 iterations before getting to 0 (try it!)

There are also numbers that require 7, 8, 9, 10, and 11 iterations (277,777,788,888,899 is apparently the smallest numbers that needs 11 iterations to get to a single digit).

Amazingly, nobody knows whether there is a starting number that will need 12 or more iterations to get to a single digit number.

Perhaps **you** will be the first person to find such a number, or *prove* that there 11 is the maximum number of iterations.

To find out more, look up:
Multiplicative Persistence
sequence A003001 in the Online Encyclopaedia of Integer Sequences
Or investigate *additive persistence* (follow the same instructions, but add the digits together rather than multiplying them)