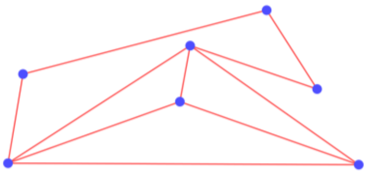
Vertices, Edges, and Regions



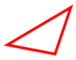
A graph

This task is all about graphs, but not graphs in the most usual sense.

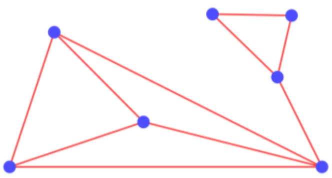
For our purposes, a graph is a diagram like the one on the left, in that it consists of:

 **vertices** (also known as *nodes*. These are the dots)

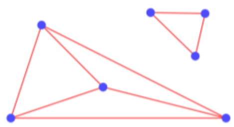
 **edges** (also known as *lines* or *links*) between the vertices, which divide the white area into

 ***regions.***

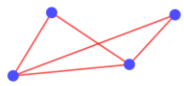
For this task, we're going to work with *connected* graphs whose edges do not cross:



A connected graph with no edges that cross



A graph that is not connected

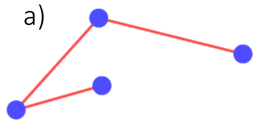


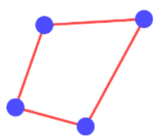
A graph whose edges cross

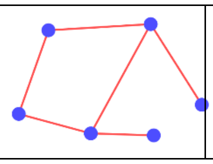
For the connected graph above, there are 7 vertices, 10 edges, and 5 regions (Note that we include the outside unbounded region)

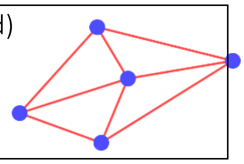
Number of vertices (V)	Number of edges (E)	Number of regions (R)
7	10	5

1. For the connected graphs (with no crossing edges) shown below, count the number of vertices, edges and regions, and gather this information into a table of results, as above.

a) 

b) 

c) 

d) 

2. Make up your own connected graphs with no crossing edges. Count the number of vertices, edges and regions, and gather this information as new rows in your table of results.
3. There is a connection between the values of V, E and R. Try to figure out what this is by looking at the numbers in your table. If you need more help, **investigate** by drawing more graphs and counting the edges, vertices and regions. It might help to focus on simple graphs.

Vertices, Edges, and Regions: Answers and Further Thoughts

1.

	Number of vertices (V)	Number of edges (E)	Number of regions (R)
(example)	7	10	5
a	4	3	1
b	4	4	2
c	6	6	2
d	5	8	5

3. I think that before revealing the answer, it would be good to give a big hint.

Big Hint: Look at the sum of V and R.

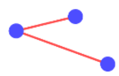
Now see if you can spot the connection.

Answer: For graphs that are connected, and have no edges crossing points, it'll always be the case that

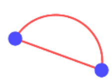
$V + R - E = 2$.

To think about *why* this ought to be the case, it might begin to make sense if you think about building up a graph by adding in vertices and edges bit by bit.

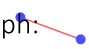
Starting with  works. We have $V = 2, E = 1, R = 1$ and so the formula . $V + R - E = 2$



If we build in **another edge with another vertex attached**, , then the value of $V + R - E$ doesn't change because R hasn't changed, and the 1 added for the increase in V cancels out with the 1 taken away for the increase in E.



If, instead, we build in **another edge without another vertex attached,** , then the value of $V + R - E$ doesn't change because V hasn't changed, and the 1 added for the increase in R cancels out with the 1 taken away for the increase in E.

The remaining question is whether all connected graphs (with no edge crossings) can be constructed by building them up in this way, from the starting graph:  .

To find out more, look up: *Euler Characteristic*