Squaring Fractions

1 Which of these three numbers is closest to 2?

$$\frac{4}{3} \times \frac{4}{3} \qquad \frac{7}{5} \times \frac{7}{5} \qquad \frac{10}{7} \times \frac{10}{7}$$

(You may use a calculator to help you to decide)

2 Can you get closer? Try to find a fraction that, when multiplied by itself, gets you closer to 2.

3 How close to 2 can you get? Keep aiming to find a fraction that *beats your personal best.*

4 Can you find a fraction that, when multiplied by itself, is *exactly* 2?

Squaring Fractions - Answers and Further Thoughts

1 $\frac{4}{3}\times\frac{4}{3}=\frac{16}{9}=1.777\ldots$ $\frac{7}{5}\times\frac{7}{5}=\frac{49}{25}=1.96$ $\frac{10}{7}\times\frac{10}{7}=\frac{100}{49}=2.0408\ldots$

So both $\frac{7}{5}\times\frac{7}{5}$ and $\frac{10}{7}\times\frac{10}{7}$ do quite well, getting only about 0.04 away from 2, but $\frac{4}{3}\times\frac{4}{3}$ isn't very close.

2 Here are examples of fractions that get very close:

$\frac{17}{12}\times\frac{17}{12}=2.0069\ldots$ $\frac{99}{70}\times\frac{99}{70}=2.000204\ldots$

3 Well done if you got even closer than those above!

4 Here's the thing: It's *impossible* to do it.

Perhaps you don't believe me.

It is possible to *prove* it, but you might need to know a bit more Maths for that. (Not a lot more though; you probably will be able to understand it before you leave school)

It is thought that the Pythagoreans about 2500 years ago were in denial about this fact. They thought that you could do everything that they wanted to with fractions. They wanted to find a fraction that, when multiplied by itself, made 2. They found that it could not be done, but treated this fact secret. Legend has it that Hippasus, one of the Pythagoreans, was murdered for stating the fact publicly.

I don't know whether this legend is true, but the story serves to demonstrate that it was, at one time anyway, a shock for humans to discover that they needed more numbers than just fractions (that is, $\frac{a\ whole\ number}{another\ whole\ number}$) to work out things that they wanted to work out.

To find out more, look up:
Irrational Numbers (this is the name for numbers that can't be written as a fraction.)
proof of irrationality of root 2
Hippasus of Metapontum